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ABSTRACT

This is one of a series of 20 booklets designed for participants in an in-service course for teachers of elementary mathematics. The course, developed by the University of Illinois Arithmetic Project, is designed to be conducted by local school personnel. In addition to these booklets, a course package includes films showing mathematics being taught to classes of children, extensive discussion notes, and detailed guides for correcting written lessons. This booklet contains: exercises on consecutive jumps, distances jumped, and competing number line rules; a summary of the problems in the film "Which Rule Wins"; and the supplement. (MK)

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THE ARITHMETIC PROJECT COURSE FOR TEACHERS

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TOPICS: Consecutive Jumps. Distances Jumped. Competing Number Line Rules.

FILM: Which Rule Wins? Grade 3

SUPPLEMENT: Computing With Positive and Negative Numbers

NAME: .....

2

SE 031 102

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BOOK TWO

I. CONSECUTIVE JUMPS WITH A NUMBER LINE RULE

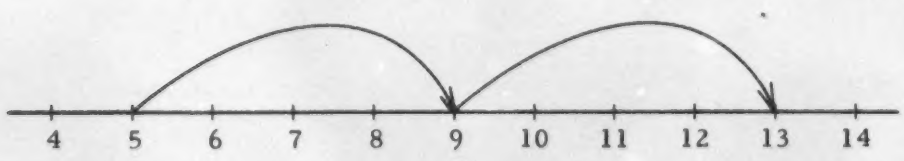
You can make several jumps, one after another, with a number line rule.

1. $\square \longrightarrow \square + 4$

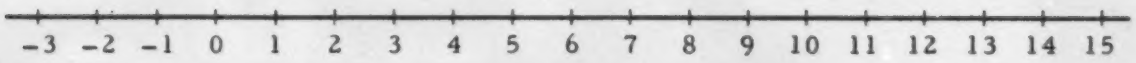
(a) Start at 5. Make one jump with this rule. Where do you land?

_____

(b) Start at 5. Make 2 consecutive jumps with this rule. (The second one starts from where the first one landed.) Where do you finally land? 13 How many spaces did you travel altogether in two jumps? _____ These jumps have been drawn on a piece of number line below.



Continue with the rule □ ⟶ □ + 4 and draw all the jumps indicated below in (c) on the following number line:



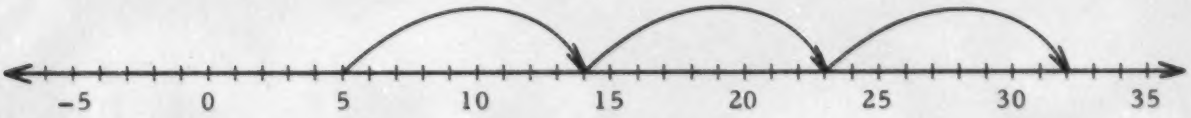
(c) Start at 0. Make one jump with this rule. Land? _____ Draw this jump. Start from your landing point and make another jump. Draw it above. Start from your second landing point. Draw another jump. Land? _____ Altogether, in these three consecutive jumps, how many spaces have you traveled from your original starting point? _____

(d) Start at 15. Make 3 consecutive jumps. Number of spaces traveled? _____ Land? _____

(e) Start at 15. Make 10 consecutive jumps. Land? _____ Number of spaces traveled? _____

(f) Start at 57. Make 25 consecutive jumps. Land? _____

2. □ ⟶ 9 + □



(a) Start at 5. Take one jump. Land? _____ Spaces moved? _____

(b) Start at 5. Take three jumps. Land? _____ Spaces moved? 27 (These jumps have been drawn above.)

(c) Start at -2. Make 4 jumps; draw them on the line above. Land? _____ Distance moved? _____

Continue with the rule $\square \longrightarrow 9 + \square$:

(d) Start at 20 . Make ten jumps. Land? _____ Spaces moved? _____

(e) Start at 100 . Make 100 jumps. Land? _____ Spaces moved? _____

(f) Starting at 100 will you ever hit 154 with this rule? _____ 260 ? _____ Do you have to find out how many jumps in order to answer these questions? _____

3. $\square \longrightarrow \square - 3$

Making consecutive jumps with a rule like $\square \longrightarrow \square - 3$ is one of several possible ways to introduce classes to negative numbers. Show jumps for (a), (b), and (c) on the number line below.

(a) Start at 10 . Draw one jump. Land? _____ Spaces moved? 3*

(b) Start at 10 . Draw three jumps. Land? _____ Spaces moved? _____

(c) Start at 10 . Draw five jumps. Land? _____ Spaces moved? _____

(d) Start at 50 . Make one jump. Land? 47 Spaces moved? _____

(e) Start at 50 . Make 20 jumps. Land? _____ Spaces moved? _____

*Notice that distance is never negative.

Continue with the rule: $\square \longrightarrow \square - 3$

(f) Start at 3. Make seven jumps. Land? _____

Spaces moved? _____

(g) Start at 3. Make 301 jumps. Land? _____

Spaces moved? _____

(h) Start at 1,000. Take 200 jumps. Land? _____

Spaces moved? _____

(i) Start at 1,000. How many jumps do you need to take to land at -2?

_____

(j) Where can you start so that after 5,000 jumps you land at 0?

_____

(k) Where can you start so that after 18 jumps you land on -18?

_____

(l) Where can you start so that after 25 jumps you land on -99?

_____

(m) Using this rule and starting at 150, give three numbers less than 80 on which you will <u>not</u> land.

_____

_____

_____

II. DISTANCES JUMPED

1. Rule: □ ⟶ □ + □ + □

(a) Start: 6 One jump. Land: _____ Distance traveled: 12

(b) Start: 5 One jump. Land: _____ Distance traveled: _____

(c) Start: -5 One jump. Land: _____ Distance traveled: 10

[Note: (-8) + (-8) + (-8) = -24]

(d) Start: 10 One jump. Land: _____ Distance traveled: _____

(e) Start: _____ One jump. Land: -12 Distance traveled: _____

(f) Start: _____ One jump. Land: _____ Distance traveled: 0

[Note: To travel 0 spaces, your landing point should be the same as your starting point.]

(g) Give a method that a child might suggest for predicting the distance traveled with any jump, given only a positive starting point.

(h) Describe what the teacher might do or say if a child gave this incorrect generalization: "The distance is three times the starting point."

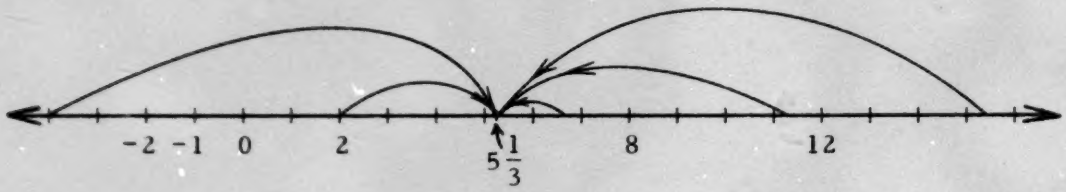
(i) On the basis of your generalization in (g), use the rule □ ⟶ 4 × □ to predict the <u>length</u> of the jump if you start at:

7 _____ 2 _____

10 _____ 0 _____

2. Rule: $\square \longrightarrow 5\frac{1}{3}$

With this rule, strange as it may seem, you land at $5\frac{1}{3}$ no matter where you start. Some jumps are shown on the line below:



(a) Start at $11\frac{1}{3}$. Land: $5\frac{1}{3}$ Spaces moved: _____

(b) Start at -1. Land: _____ Spaces moved: _____

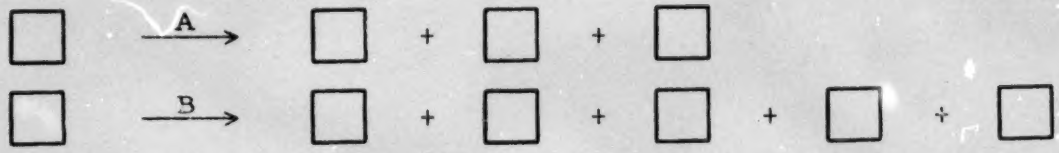
(c) Start at -1 and make twelve consecutive jumps. Land: _____

(d) You want to travel 10 spaces. At which two places on the line can you start? _____, _____

(e) Name a place to start so that you travel 100 spaces or more to the right. _____

III. COMPETING NUMBER LINE RULES

Suppose you have two jumping rules



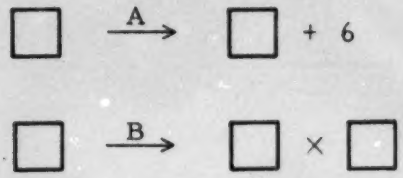
and your starting point for each is $17\frac{3}{7}$. If you make one jump with each rule, which rule gives you the larger number to land on? You can tell that the answer is rule B without tedious computation.

But rule B doesn't win for all starting numbers. If you start at -6, rule A gives you the larger number because -18 is larger than -30.

1. Complete the following chart:

	Starting number	Which rule wins?
(a)	100	
(b)	$\frac{1}{100}$	
(c)	-1	
(d)	$-4\frac{3}{11}$	
(e)		TIE

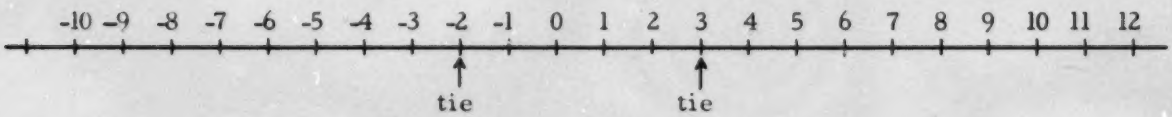
2. Now consider these two rules:



(a) If you start at 1 with rule A and rule B, which one wins? _____

Which one wins if you start at 4? _____ Where is a starting point so that they tie? _____

(Note that -2 also is a starting point at which they tie, because $-2 \times -2 = +4$ and $-2 + 6 = +4$.)



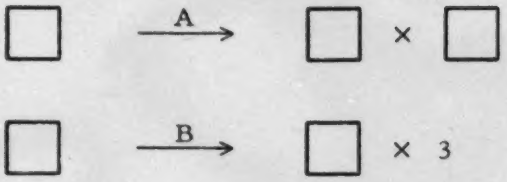
(b) Which rule wins if you start at $3\frac{1}{10}$? _____

Which rule wins if you start at -5? _____

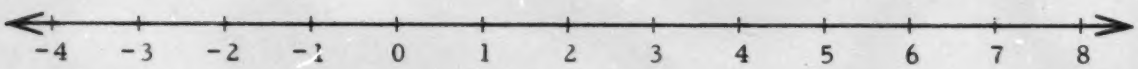
If you start at 0? _____ At $-7{,}134$? _____

At $-1\frac{1}{11}$? _____

3. Now the two competing rules are:



Write a few questions (and give the answers) that your students should work so that they can learn to predict all tie points and the winning rule for any starting point. (If you teach first or second grade, you may, if you wish, write problems concerning □ $\xrightarrow{C}$ □ + □ versus □ $\xrightarrow{D}$ □ + 4. Indicate which set of rules you are using.)



4. Write a generalization that one of your students gave or might give about which rule in problem #3 wins.

Epilogue

Why is distance never negative?

The fact that distance is never negative comes not as mindless fiat from the mathematical community but rather as an attempt to reflect common intuitions about distance. Certainly we want the distance between New York and Boston to be the same as the distance between Boston and New York; indeed, if one of these distances were the negative of the other then a round trip would require traveling a distance of zero miles. One can imagine the following conversation:

"How far did you drive last weekend?"

"Zero miles."

"But I thought you went to New York!"

"I did."

"How far is it to New York?"

"About 200 miles."

"And back?"

"The same, of course."

"Then how far <u>did</u> you drive?"

"I didn't. I flew."

Some teachers may wonder if there are any other things to remember about distance. For the sake of completeness, everything that anyone needs to know about distance is listed below:

1. Distance is never negative.
2. If A and B are any two points, the distance between A and B is the same as the distance between B and A .
3. The distance between any point and itself is zero.
4. The distance between any two different points is not zero. (And since it can't be negative, it must be positive.)
5. If A, B and C are any three points, the sum of the distances from A to B and from B to C is no less than the distance from A to C . Thus the trip from Boston to New York cannot be shortened by visiting any intermediate city (Providence or Cleveland, for example).

Anything that has these properties can rightfully be called distance, and, conversely, any notion of distance must satisfy these properties.

It may be worthwhile to mention that the distance between 4 and 9 is not the same thing as the jump from 4 to 9. The distance between 4 and 9 is just a number (5) whereas the jump involves a starting number and a landing number.

Summary of Problems in the Film
"Which Rule Wins?"

3rd Grade, Cunniff School, Watertown, Massachusetts
Teacher: Phyllis Klein

Which number is bigger? 3 or 1
1 or 0
1 or -4
1 or -10
1 or -99
100 or 0
100 or -100

We'll use two rules:

Rule a: $\square \xrightarrow{a} \square + \square$

Rule b: $\square \xrightarrow{b} \square + 6$

Use rule a; start at 10. (Land: 20)

Use rule b; start at 10. (Land: 16)

Which rule gives the bigger number?

Start at 2 with each rule; which rule gets you to the bigger number?

Give me another starting place where rule b wins. (5)

At 5, by how much does rule b win?

What is another starting place where rule b wins? (3)

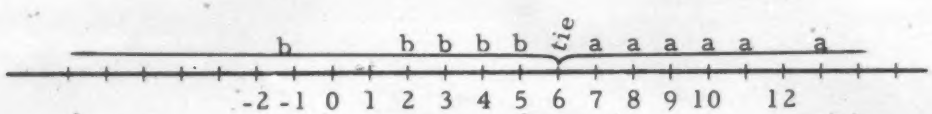
By how much will rule b win at 3?

Give me a number where rule a wins. (9), (12)

What is the tie point? (6)

Which rule wins at 7? At $8\frac{1}{2}$? At $7\frac{2}{3}$?

Give me an interesting number where rule b wins. $(-1\frac{1}{3})$



We'll change rule b so that the rules are:

$\square \xrightarrow{a} \square + \square$

$\square \xrightarrow{b} \square + 3$

Give me an interesting place to start. (2)

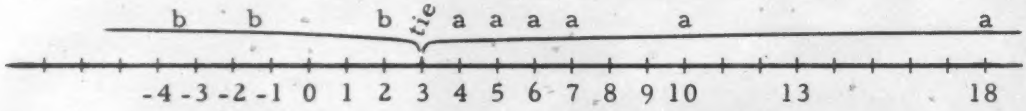
Which rule wins at 2?

Which rule wins at 20?

Which rule wins at 10?

Where will the tie be?

Tell me all the places where rule a will win.



We'll change rule b again:

$\square \xrightarrow{a} \square + \square$

$\square \xrightarrow{b} \square + 17\frac{1}{2}$

What would the tie be?

Here are some new rules:

$\square \xrightarrow{a} \square + 7$

$\square \xrightarrow{b} \square + \square + \square$

Which rule will win at 0?

Which rule will win at 1?

Find a tie. Between which numbers do you look for it?

What would happen at 5? at 6? at 7? Which rule would win?

Which rule wins at 4? By how much?

Which rule wins at 3? By how much?

What is the tie point? $(3\frac{1}{2})$

What would happen if we start a little above $3\frac{1}{2}$? Would rule a or rule b win?

If we go to the left of $3\frac{1}{2}$, which rule wins?

Here are some new rules:

$\square \xrightarrow{a} \square \div 11$

$\square \xrightarrow{b} \square + \square + \square$

Which rule wins at 5?

Which rule wins at 6?

What is the tie?

New Rules:

$\square \xrightarrow{a} \square + 300$

$\square \xrightarrow{b} \square + \square + \square$

What is the tie point?

Now we'll change rule a again:

$\square \xrightarrow{a} \square + 4$

$\square \xrightarrow{b} \square + \square + \square$

What is the tie?

Change rule a again:

$\square \xrightarrow{a} \square + 50$

$\square \xrightarrow{b} \square + \square + \square$

What is the tie point?

Another rule a:

$\square \xrightarrow{a} \square + 52$

$\square \xrightarrow{b} \square + \square + \square$

What is the tie point now?

Rule a is changed again:

$\square \xrightarrow{a} \square + 51$

$\square \xrightarrow{b} \square + \square + \square$

Tie point?

17

New rules:

$\square \xrightarrow{a} \square + \square$

$\square \xrightarrow{b} \square + \square + \square$

Where do they tie?

Give me a number where rule b wins. (5)

By how much does rule b win at 5?

Give another number where rule b wins. (2)

By how much does rule b win?

What number should I start at so that rule b would win by 17?

Tell me all the numbers where rule b wins.

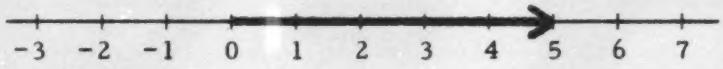
How about $7\frac{1}{2}$? 100 ?

Tell me all the numbers where rule a wins.

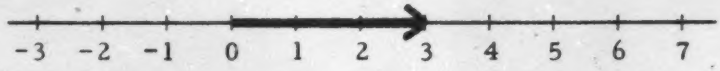
Supplement
Computing With Positive and Negative Numbers

One way of viewing addition is by using arrows on the number line. In doing the problem $5 + 3$, for instance, we first consider two arrows: the one from 0 to 5 and the one from 0 to 3.

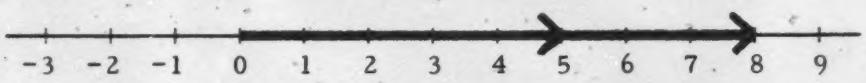
The 5 arrow:



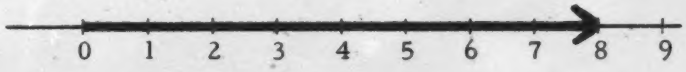
The 3 arrow:



Adding 5 and 3 means to combine these two arrows in the following way. We slide the second one so that its tail coincides with the head of the first arrow:

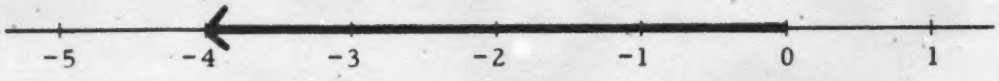


Now we look at the resulting arrow

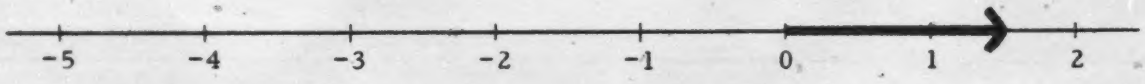


to get the answer: 8.

Try a harder one——what is $-4 + 1\frac{1}{2}$? The arrows we combine are



and

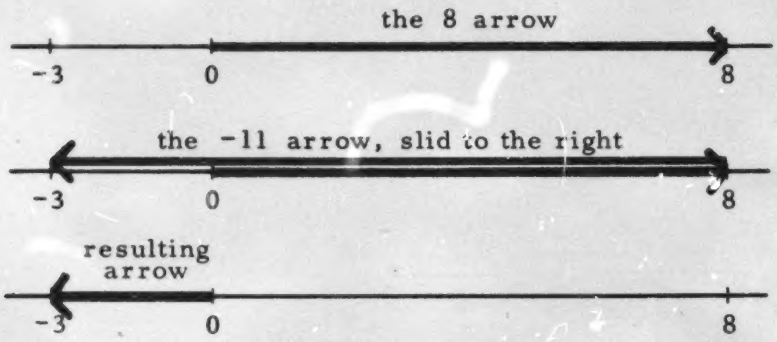


Sliding the second one, we get

and the resultant arrow is

which corresponds to $-2\frac{1}{2}$. So $-4 + 1\frac{1}{2} = -2\frac{1}{2}$

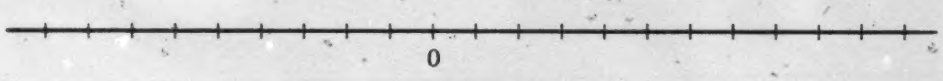
One more example: 8 + −11 would be shown like this:



Answer: −3

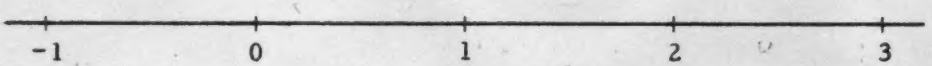
1. Draw on the number line the arrows to find the answer to the following:

$7\frac{1}{2} + (-6\frac{1}{2}) =$ _____



2. Draw on the number line the arrows to find the answer to the following:

$-\frac{1}{3} + \frac{5}{3} =$ _____

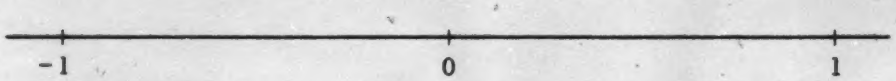


3. Draw on the number line the arrows to find the answer to the following:

$-2 + (-3) =$ _____

4. Draw on the number line the arrows to find the answer to the following:

$-\frac{1}{3} + \frac{1}{6} =$ _____



For the following problems, draw arrows if you need to. An assortment of number lines is provided for you at the bottom of the page.

5. $3\frac{1}{2} + -3 =$ _____

6. $4 + -3\frac{1}{2} =$ _____

7. $6 + -6 =$ _____

8. $7\frac{3}{8} + -7\frac{3}{8} =$ _____

9. $-4 + 3\frac{1}{2} =$ _____

10. $-4 + -3\frac{1}{2} =$ _____

11. $-10\frac{1}{10} + 10 =$ _____

12. $5 + 3 + -9 =$ _____

13. $5 + 3 + -5 =$ _____

14. $5 + 2\frac{2}{7} + -2\frac{2}{7} =$ _____

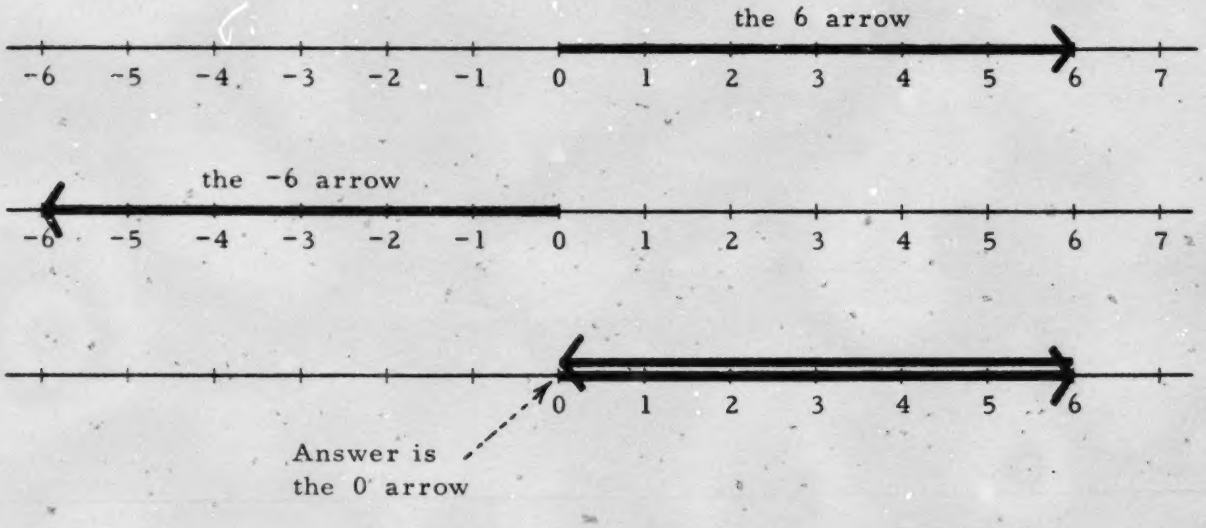
15. $-100 + 90 =$ _____

16. $-104 + 107 =$ _____

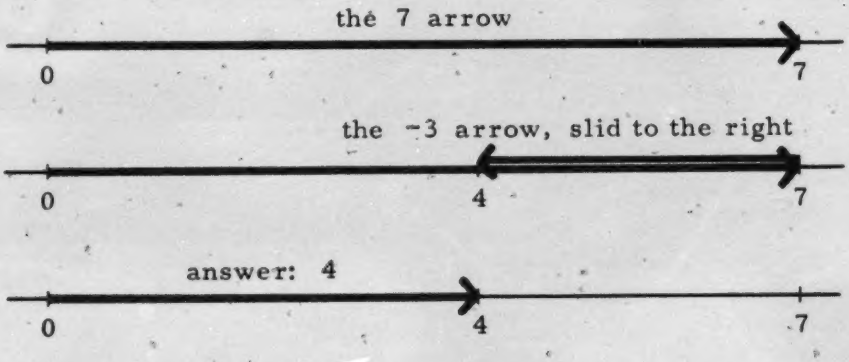
For each arrow there is an opposite arrow——the arrow of the same length but of the opposite direction. It is clear that the opposite of the 3 arrow is the -3 arrow, the opposite of the $-73\frac{1}{2}$ arrow is the $73\frac{1}{2}$ arrow, and so on. The opposite of the 0 arrow is itself, since there is no direction to worry about.

The sum of any arrow and its opposite is the zero arrow. Example:

$6 + -6$

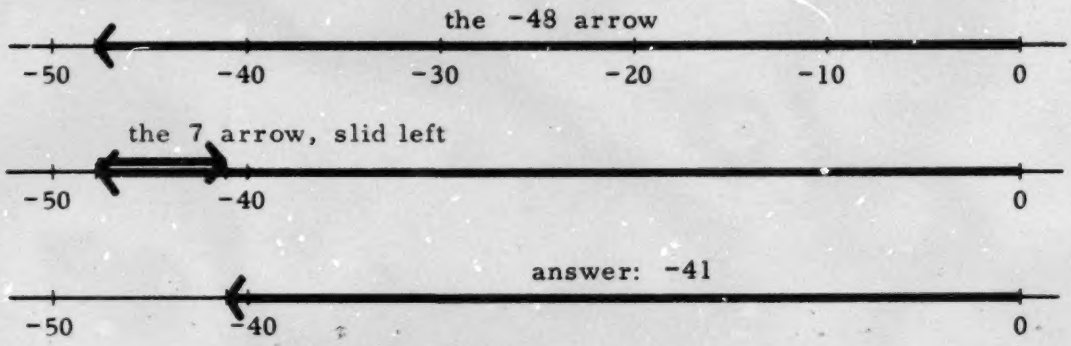


We can now <u>define</u> subtraction as follows: If □ and △ are numbers, then □ − △ shall mean □ + the opposite of △. Example: $7 - 3$ means $7 +$ the opposite of 3, i.e., $7 + -3$. If you do this with arrows on the number line you get the 4 arrow:



(It's good to see that this definition of subtraction agrees with what you and your pupils already know.)

Another one: $-48 - (-7)$. The opposite of the -7 arrow is the 7 arrow. We can rewrite the problem as $-48 + 7$.



More problems:

1. $-7 - (-7) =$ _____

2. $-7 - (-6) =$ _____

3. $-7 - (-8) =$ _____

4. $-7 + (-8) =$ _____

5. $3\frac{1}{2} - (-3\frac{1}{2}) =$ _____

6. $100 - (-101) =$ _____

7. $100 - 101 =$ _____

8. $100 - (-101) + 5 =$ _____

9. $(10 - (-2)) - (-3) =$ _____

10. $(10 - (-2)) + (-3) =$ _____

11. $-7\frac{1}{2} - 7\frac{1}{2} - (-7\frac{1}{2}) =$ _____

12. $-11\frac{1}{3} - (-11\frac{1}{3}) + 12 =$ _____

ANSWERS

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1. 1
2. $\frac{4}{3}$
3. -5
4. $-\frac{1}{6}$

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5. $\frac{1}{2}$
6. $\frac{1}{2}$
7. 0
8. 0
9. $-\frac{1}{2}$
10. $-7\frac{1}{2}$
11. $-\frac{1}{10}$
12. -1
13. 3
14. 5
15. -10
16. 3

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1. 0
2. -1
3. 1
4. -15
5. 7
6. 201
7. -1
8. 206
9. 15
10. 9
11. $-7\frac{1}{2}$
12. 12

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